Digital Communication Systems

ECS 452

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8. Optimal Detection for Additive Noise Channels

1-D Case

Office Hours:

BKD, 6th floor of Sirindhralai building

Monday	10:00-10:40
Tuesday	12:00-12:40
Thursday	14:20-15:30

Review: MAP decoder

3.41. A recipe for finding the MAP decoder (optimal decoder) and its corresponding error probability:

(a) Find the **P** matrix by scaling elements in each row of the **Q** matrix by their corresponding prior probability $p(x)$.

(b) Select (by circling) the maximum value in each column (for each value of y) in the **P** matrix.

- If there are multiple max values in a column, select one. This won't affect the optimality of your answer.

(i) The corresponding x value is the value of $\hat{x}$ for that y.

(ii) The sum of the selected values from the **P** matrix is $P(\mathcal{C})$.

(c) $P(\mathcal{E}) = 1 - P(\mathcal{C})$.

Review: MAP decoder

Example 3.43. Find the MAP decoder and its corresponding error probability for the DMC channel whose $\mathbf{Q}$ matrix is given by

$$\mathbf{Q} = \begin{array}{c c} x \setminus y & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.3 & 0.4 & 0.3 \end{bmatrix} \end{array} \xrightarrow[\times 0.4]{\times 0.6} \begin{array}{c c} x \setminus y & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \hat{x} & \begin{array}{ccc} 0 & 1 & 0 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 0.30 & 0.12 & 0.18 \\ 0.12 & 0.16 & 0.12 \end{bmatrix} \end{array} = \mathbf{P}$$

y	$\hat{x}(y)$
1	0
2	1
3	0

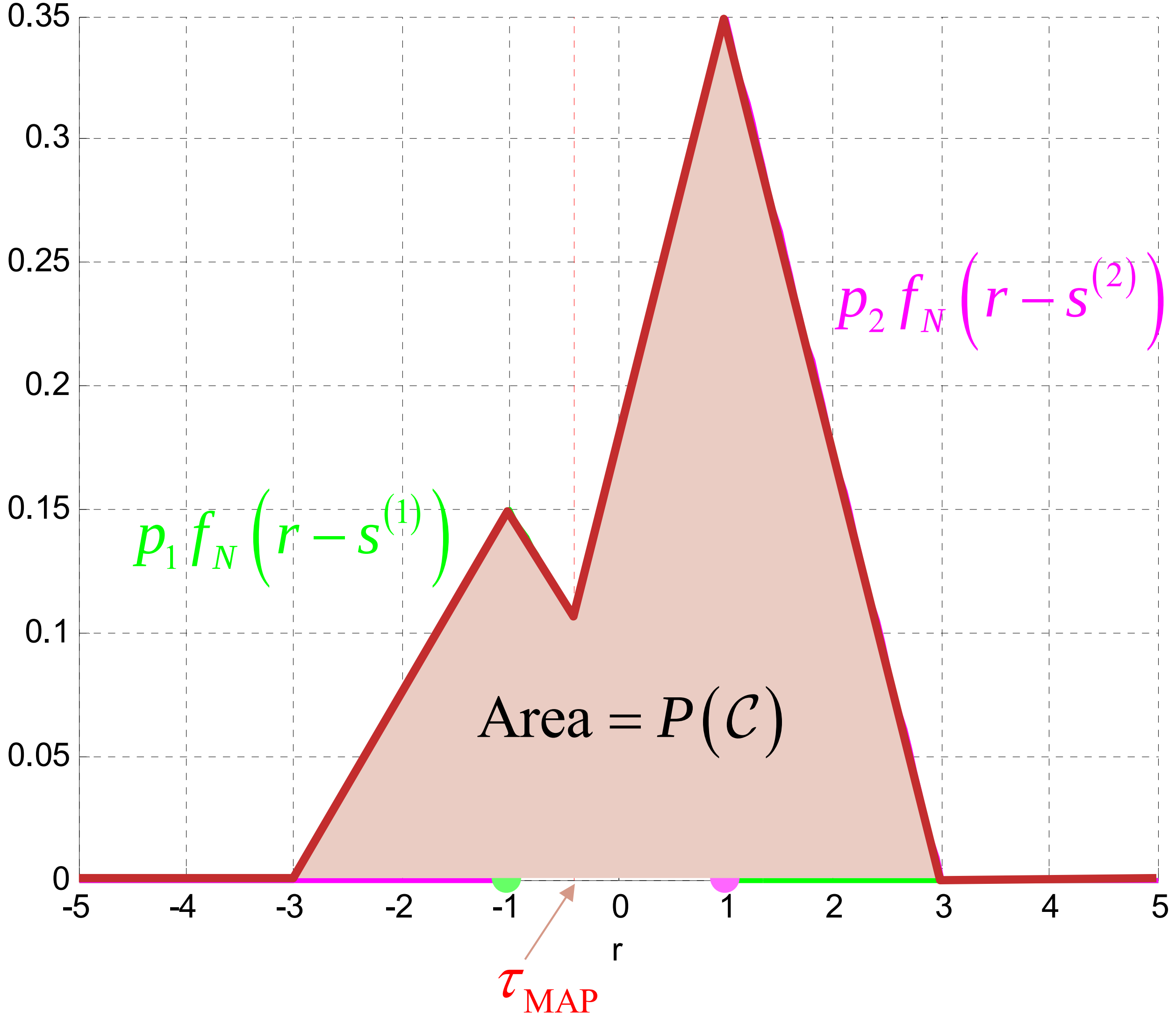
and $\underline{\mathbf{p}} = [0.6, 0.4]$. Note that the DMC is the same as in Example 3.26 but the input probabilities are different.

$P(\mathcal{C}) = 0.30 + 0.16 + 0.18 = 0.64$

$P(\mathcal{E}) = 1 - 0.64 = 0.36$

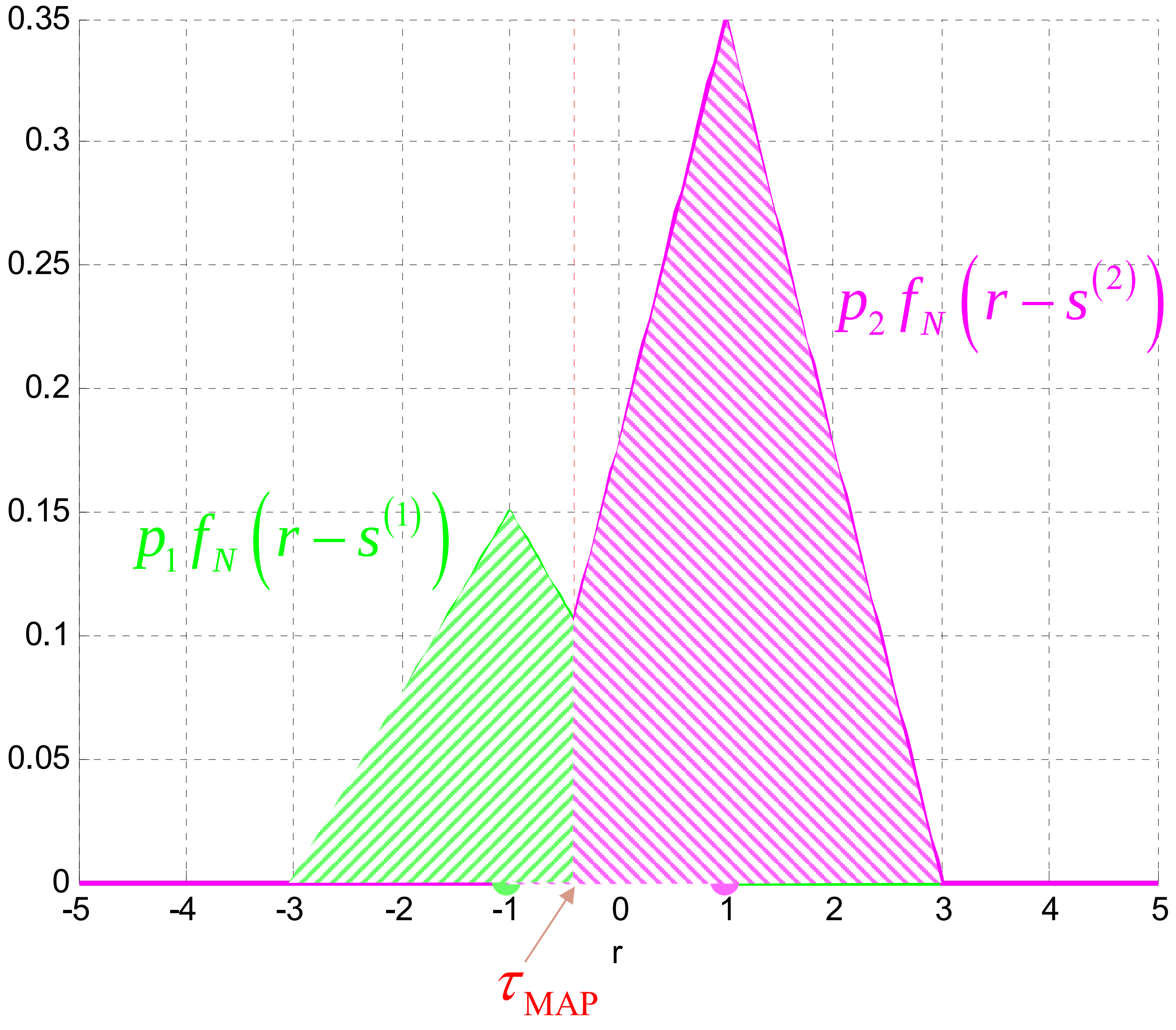
Error Probability

Ex. Binary PAM under "Triangular" Noise



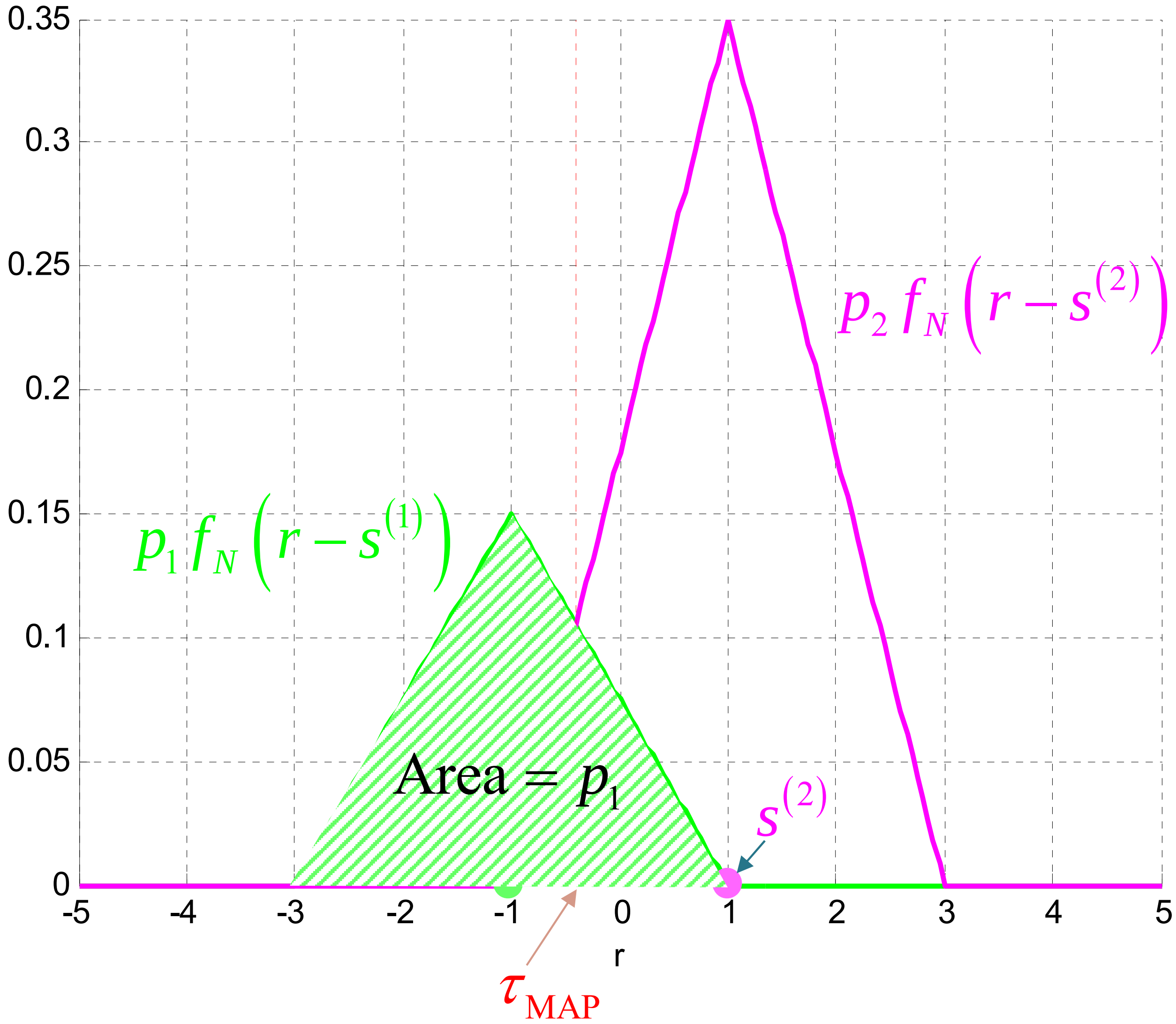
Error Probability

Ex. Binary PAM under "Triangular" Noise



Error Probability

Ex. Binary PAM under "Triangular" Noise



Error Probability

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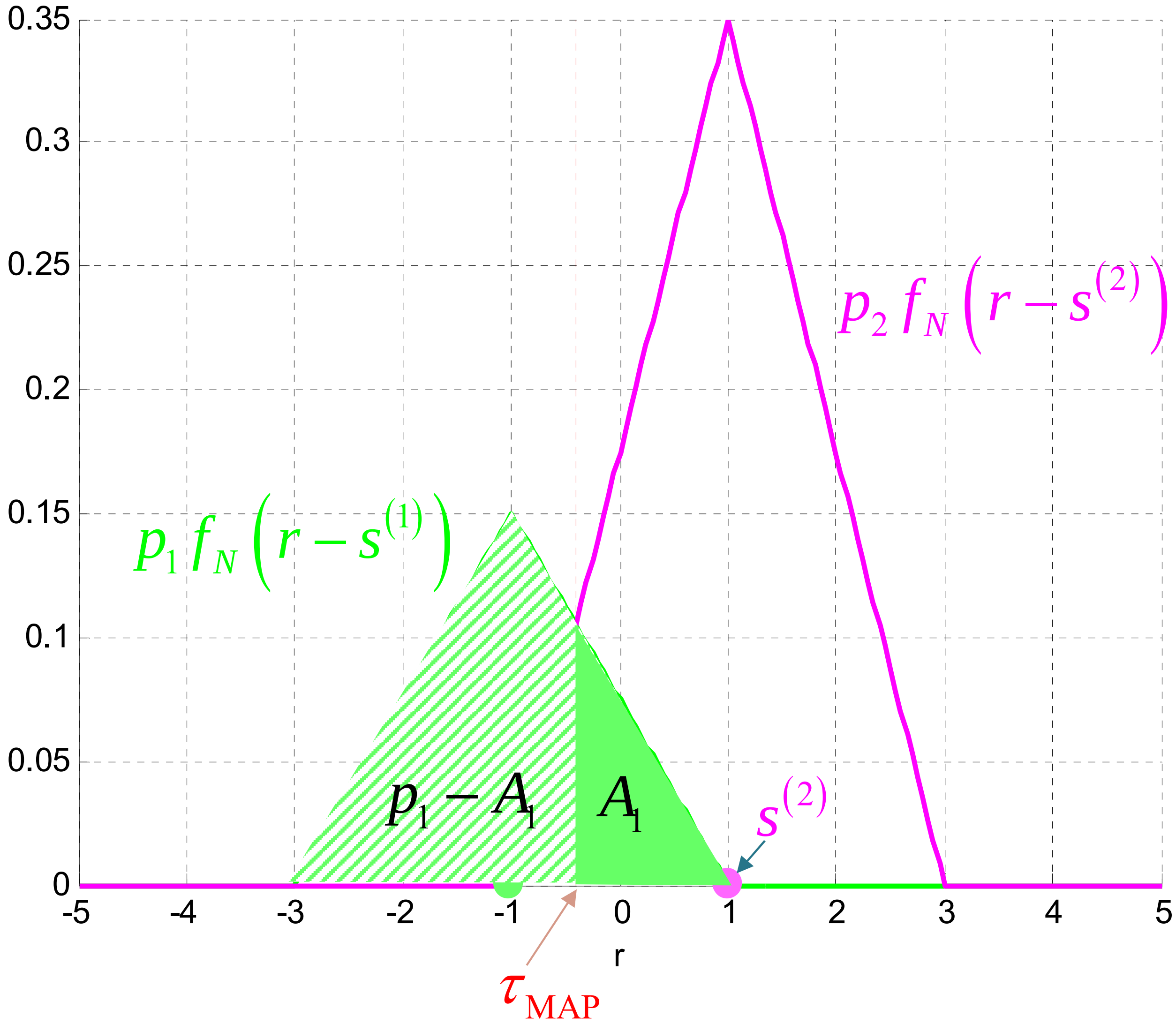

Error Probability

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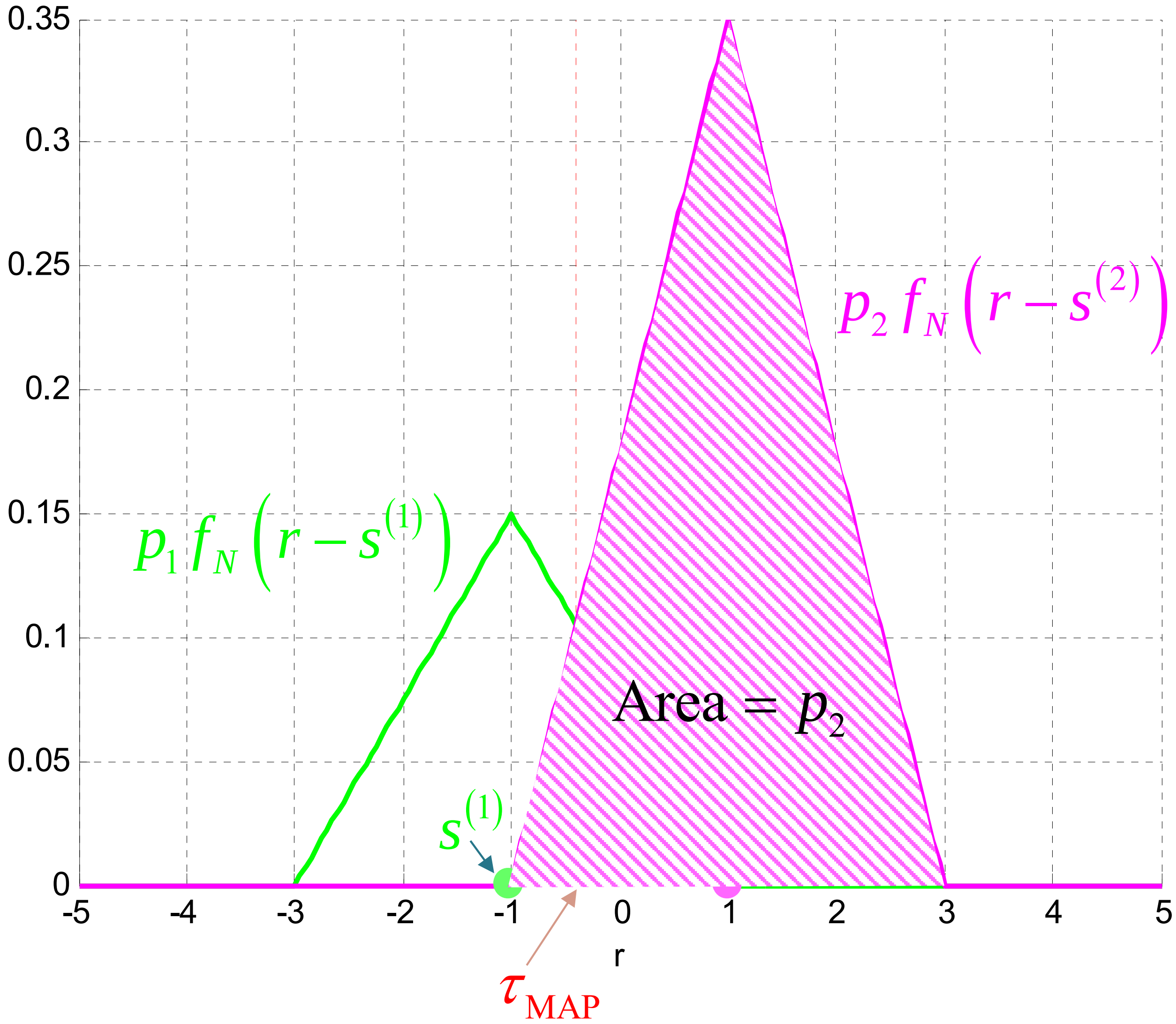
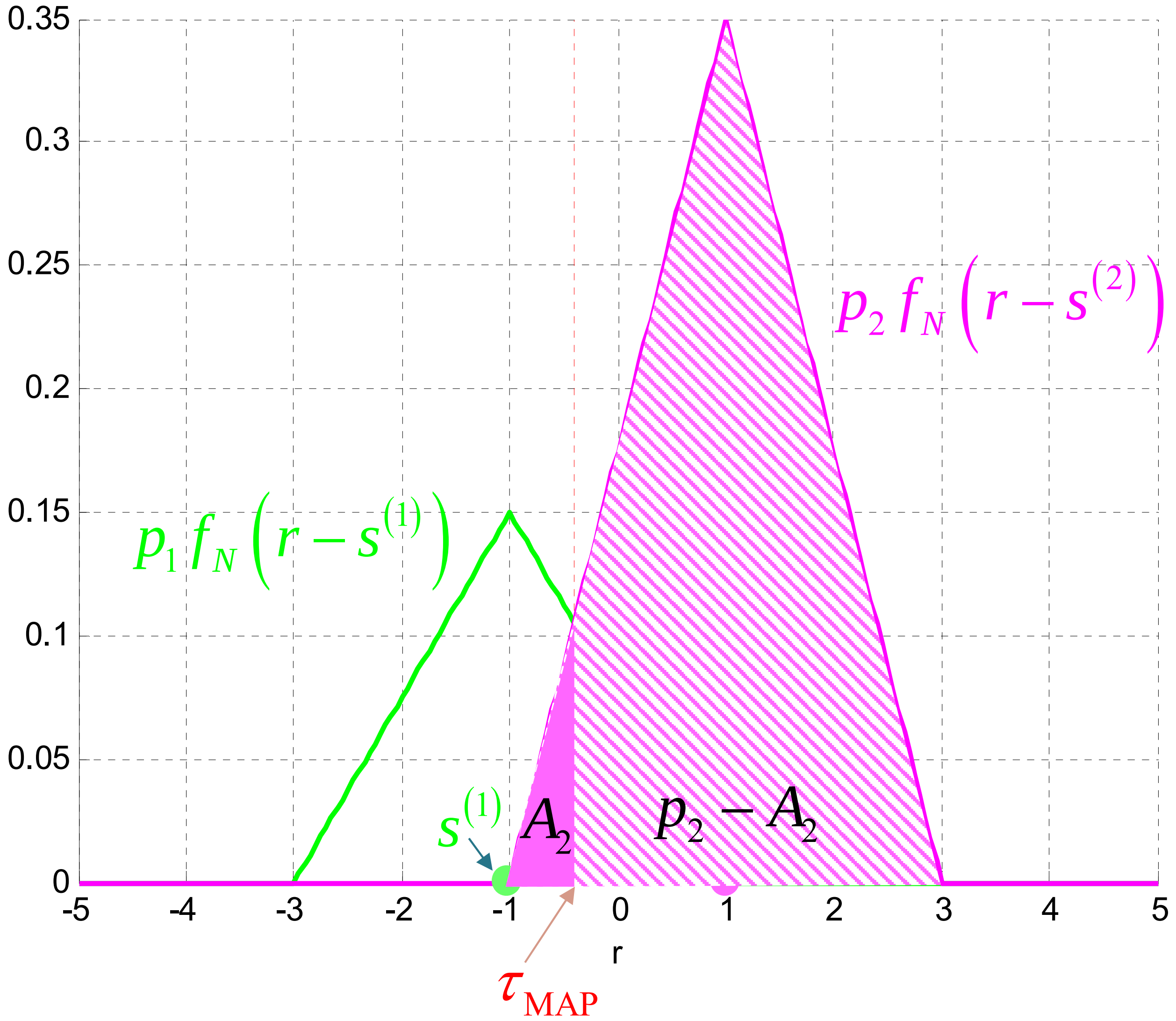


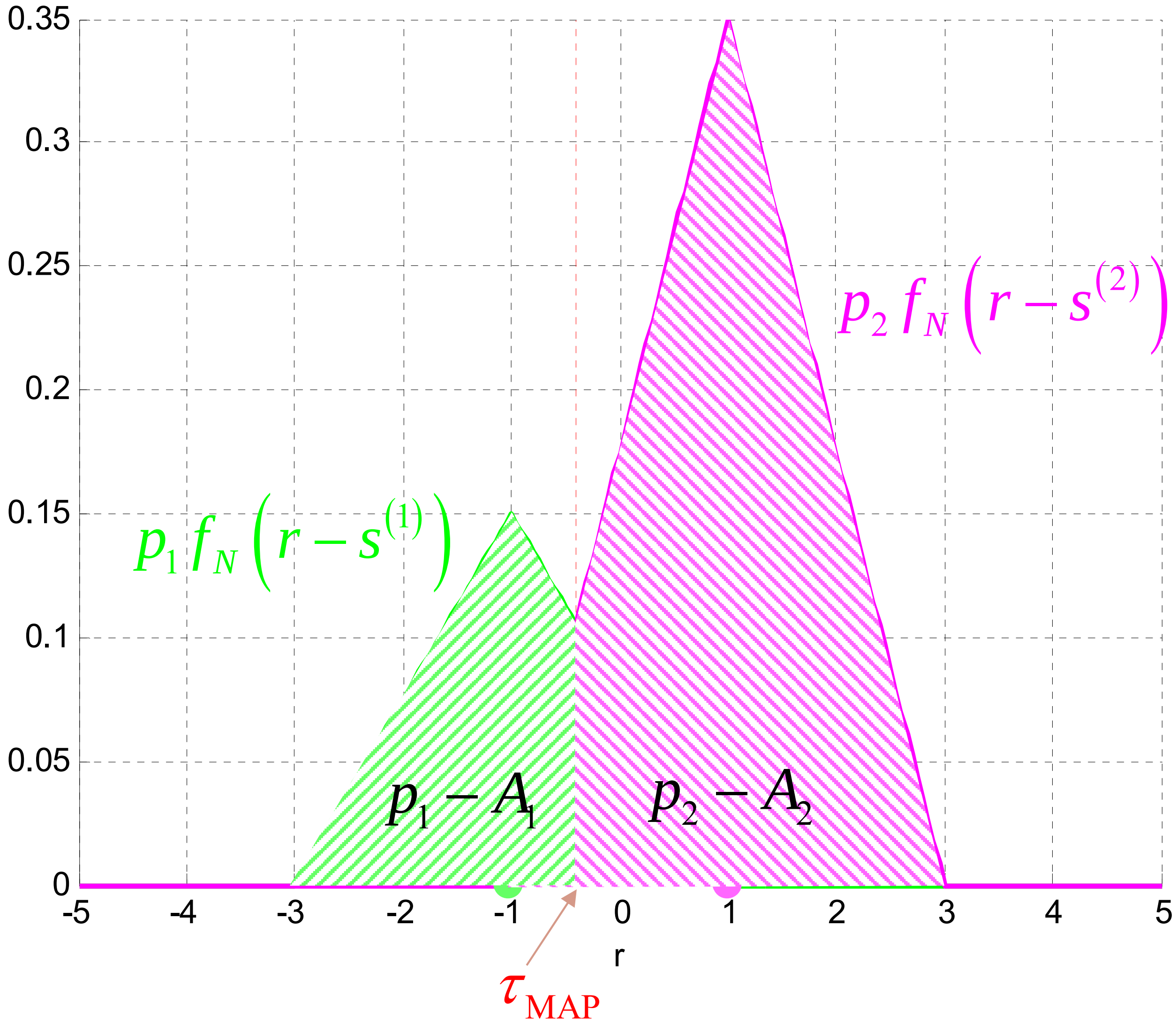
Error Probability

Ex. Binary PAM under "Triangular" Noise



Error Probability

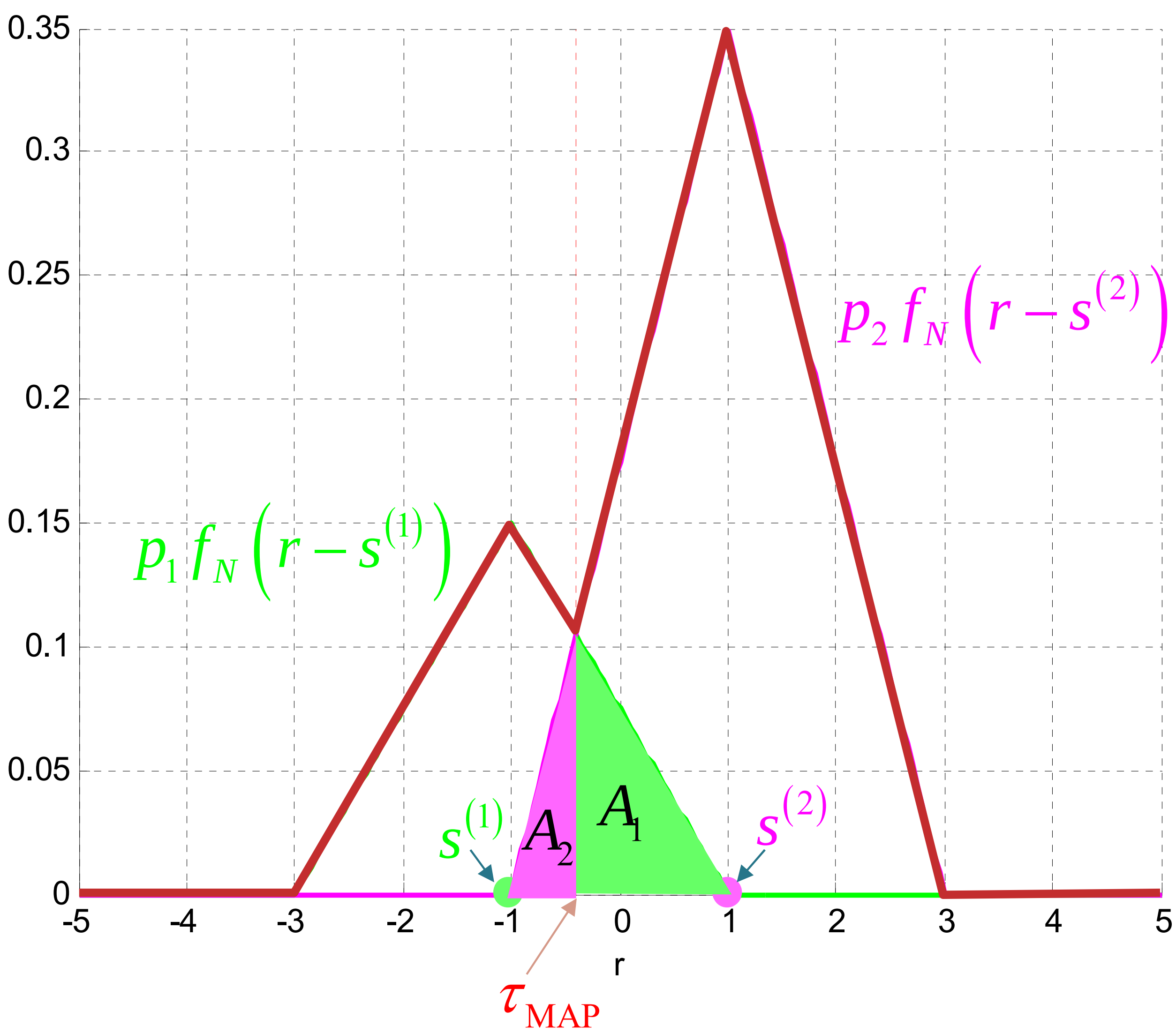
Ex. Binary PAM under "Triangular" Noise



$$P(\mathcal{C}) = (p_1 - A_1) + (p_2 - A_2)$$
$$= 1 - (A_1 + A_2)$$
$$P(\mathcal{E}) = 1 - P(\mathcal{C})$$
$$= A_1 + A_2$$

Error Probability

Ex. Binary PAM under "Triangular" Noise



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Error Probability

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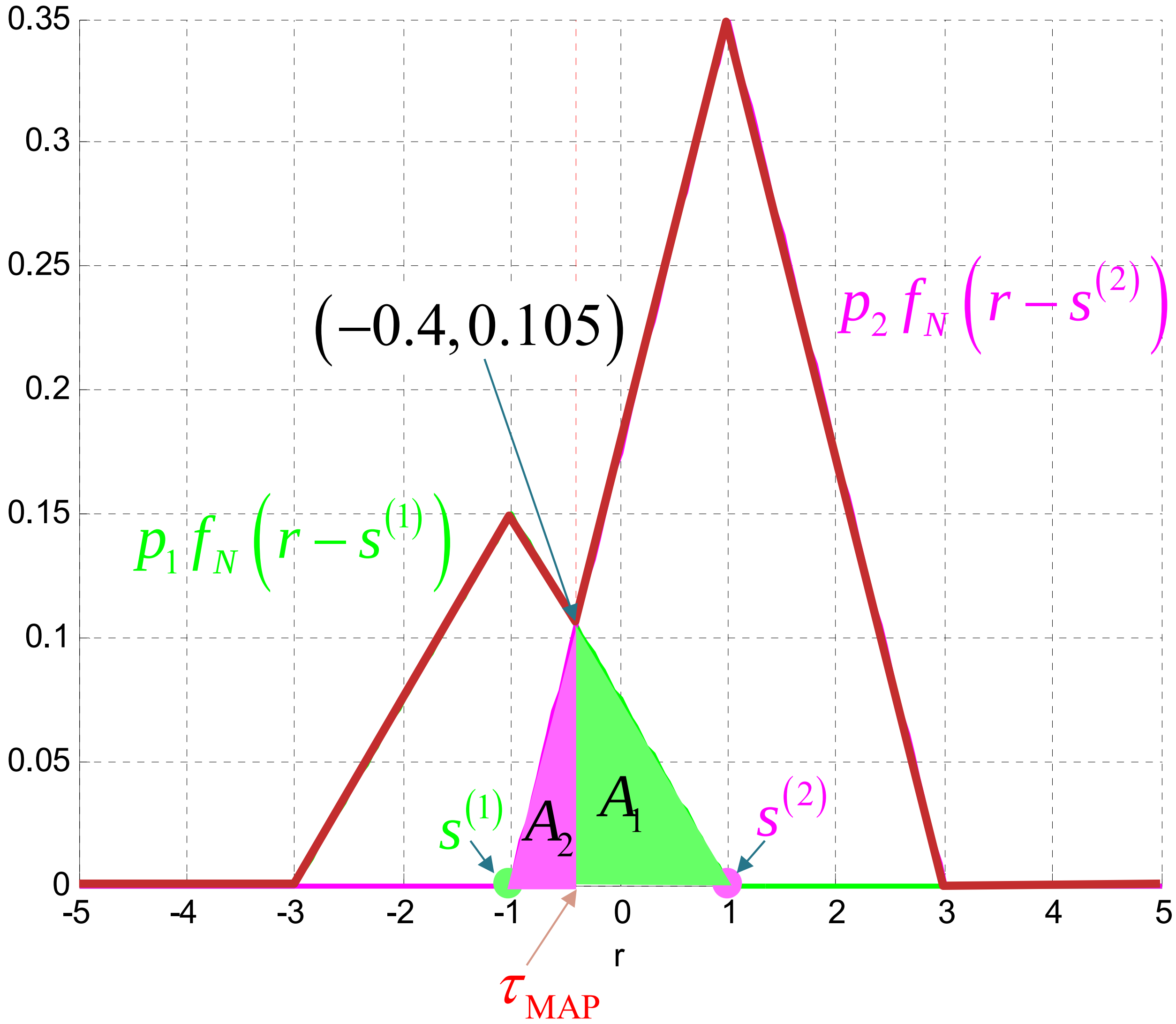

$$
\begin{aligned}
P(\mathcal{C}) &= (p_1 - A_1) + (p_2 - A_2) \\
&= 1 - (A_1 + A_2)
\end{aligned}
$$

$$
\begin{aligned}
P(\mathcal{E}) &= 1 - P(\mathcal{C}) \\
&= A_1 + A_2 \\
&= \frac{1}{2} \times 2 \times 0.105 \\
&= 0.105
\end{aligned}
$$